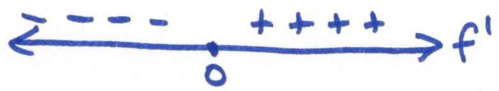
Mr. Michael T. Davis Calculus	Sections 5.1 Practice Quiz March 18, 2017

Name: Solutions

1. Determine the extreme values of the function $f(x)=x^2$ Domain: $(-\infty,\infty)$

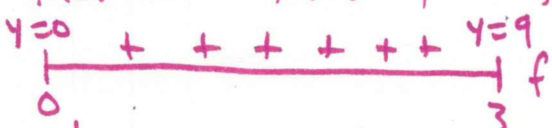
$f'(x)=2x$ $f'(x)=0$ $0=2x$ $x=0$ S.P. $f(0)=0$



f(x) has an abs. min of y=0 at x=0 since f' changes from Neg. to. Pos. at x=0.

2. Determine the extreme values of the function $f(x)=x^2$ on $[0,3]$

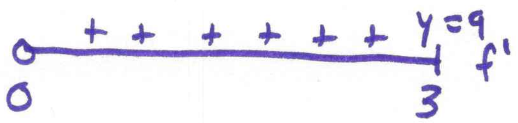
$f(0)=0$ $f(3)=9$ $f'(x)=2x$ $f'(x)=0$ $0=2x$ $x=0$



f(x) has an abs. min. of y=0 at x=0 and an abs. max. of y=9 at x=3 since $f'(x)>0$ on $(0,3)$.

3. Determine the extreme values of the function $f(x)=x^2$ on $(0,3]$

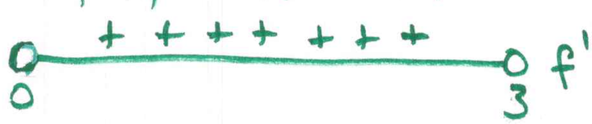
$f(3)=9$ $f'(x)=2x$ $f'(0)=0$



f(x) has no min. value and f(x) has an. abs. max. of y=9 at x=3 since $f'(x)>0$ on $(0,3)$

4. Determine the extreme values of the function $f(x)=x^2$ on $(0,3)$

$f'(x)=2x$ $f'(0)=0$

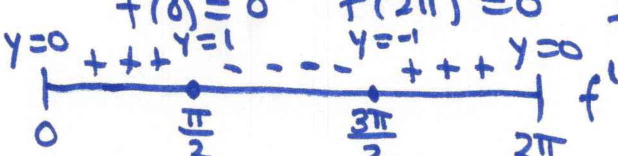


f(x) has no min. value nor does f(x) have a max. value.

5. Determine the extreme values of the function $f(x)=\sin(x)$ on $[0,2\pi]$

$f(0)=0$ $f(2\pi)=0$ $f'(x)=\cos x$ $f'(\frac{\pi}{2})=0$ $f'(\frac{3\pi}{2})=0$

$f(\frac{\pi}{2})=1$ $f(\frac{3\pi}{2})=-1$



f(x) has a rel. min. of y=0 at x=0, an abs. max. of y=1 at $x=\frac{\pi}{2}$, an abs. min. of y=-1 at $x=\frac{3\pi}{2}$, and a rel. max. of y=0 at $x=2\pi$

6. Determine the extreme values of the function $f(x)=\cos(x)$ on $[0,2\pi]$

$f(0)=1$ $\quad f(2\pi)=1$ $\quad f'(x)=-\sin x$ $\quad f'(0)=-\sin 0=0,$

$f'(\pi)=-\sin\pi=0,$ $\quad f'(2\pi)=-\sin(2\pi)=0$

$y=1$ $\quad - - - -$ $\quad y=-1$ $\quad + + + +$ $\quad y=1$ $\quad f'$

$0 \quad \pi \quad 2\pi$

$f(x)$ has an abs. max. of $y=1$ at $x=0$ and at $x=2\pi$. $f(x)$ has an abs. min. of $y=-1$ at $x=\pi$ since f' changes from Neg to Pos.

7. Determine the extreme values of the function $f(x)=\tan(x)$ on $[0,2\pi]$

$f(0)=\tan 0=0$ $\quad f(2\pi)=\tan 2\pi=0$

$f\left(\frac{\pi}{2}\right)=$ undefined $\quad f\left(\frac{3\pi}{2}\right)=$ undefined

$f'(0)=1$

$f'(2\pi)=1$

$f'(x)=\sec^2 x=\dfrac{1}{\cos^2 x}$ $\quad f'(x)\neq 0$ $\quad f'(x)=$ undefined at $x=\frac{\pi}{2}$ and $x=\frac{3\pi}{2}$

$y=0$ $\quad + +$ $\quad + + + +$ $\quad + +$ $\quad y=0$ $\quad f'$

$0 \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad 2\pi$

$f(x)$ has a rel. min of $y=0$ at $x=0$ and a rel. max. of $y=0$ at $x=2\pi$

8. Determine the extreme values of the function $f(x)=\sqrt{x}$ on $[0,16]$

$f(0)=0$ $\quad f(16)=4$ $\quad f'(x)=\frac{1}{2}x^{-\frac{1}{2}}=\dfrac{1}{2\sqrt{x}}$

$f'(0)=$ undefined

$y=0$ $\quad + + + + + + +$ $\quad y=4$ $\quad f'$

$0 \quad 16$

$f(x)$ has an abs. min. of $y=0$ at $x=0$ and an abs. max of $y=4$ at $x=16$ since $f'>0$ on $(0,16)$

9. Determine the extreme values of the function $f(x)=\dfrac{1}{x}$

Domain: $(-\infty,0)\cup(0,\infty)$ $\quad f'(x)=\dfrac{-1}{x^2}$

$- - - - - - -$ $\quad - - - - -$ $\quad f'$

0

$f(x)$ has no min. or max. values.

10. Determine the extreme values of the function $f(x)=x^{\frac{4}{3}}$

Domain: $(-\infty,\infty)$ $\quad f'(x)=\frac{4}{3}x^{\frac{1}{3}}$ $\quad f'(0)=0$

$- - - - -$ $\quad y=0$ $\quad + + + + +$ $\quad f'$

0

$f(x)$ has an abs. min. of $y=0$ at $x=0$ since $f'(x)$ changes from Neg. to Pos.

11. Determine the extreme values of the function $f(x)=x^{\frac{2}{3}}$

Domain: $(-\infty,\infty)$

$f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3\sqrt[3]{x}}$

$f'(0)=$ undefined $x=0$ C.P.

$f(0)=0$

$- - - - \quad 0 \quad + + + +$ f'

$f(x)$ has an abs. min. of $y=0$ at $x=0$ since $f'(x)$ changes from Neg. to Pos.

12. Determine the extreme values of the function $f(x)=e^{x}$

Domain: $(-\infty,\infty)$

$f'(x)=e^{x}$ $f'(x)\neq 0$ $f'(x)\neq$ undefined

There are no C.P. or S.P. or endpoints

$+ + + + + + +$ f

$f'(x)>0$ $e^{x}>0$

There are no extreme values.

13. Determine the extreme values of the function $f(x)=\frac{x}{x-1}$

Domain: $\mathbb{R}$, $x\neq 1$

$f'(x)=\frac{(x-1)(1)-x(1)}{(x-1)^2}=\frac{-1}{(x-1)^2}$

$f'(x)\neq 0$ $f'(1)=$ undefined

$x=1$ is not a C.P. because $x=1$ is not in the domain of $f(x)$

$- - - - \quad 1 \quad - - - -$ f'

There are no extreme values.

14. Determine the extreme values of the function $f(x)=\frac{1}{\sqrt{4-x^2}}$

Domain: $(-2,2)$

There are no endpoints

$f'(x)=\frac{\sqrt{4-x^2}(0)-1\cdot\frac{1}{2}(4-x^2)^{-\frac{1}{2}}(-2x)}{4-x^2}$

$=\frac{x}{\sqrt{4-x^2}\,(4-x^2)}$

$-2 \quad - - - - \quad 0 \quad + + + + \quad 2$ f'

$f'(0)=0$ $f'(x)\neq$ undefined

$f(0)=\frac{1}{2}$

$f(x)$ has an abs. min. of $y=\frac{1}{2}$ at $x=0$ since $f'(x)$ changes from Neg to Pos.

15. Determine the extreme values of the function $f(x)=\frac{1}{x}+\ln(x)$ on $0.5\leq x\leq 4$

$f(0.5)=2+\ln(\frac{1}{2})\approx 1.3069$

$f(4)=\frac{1}{4}+\ln 4\approx 1.6363$

$f'(x)=\frac{-1}{x^2}+\frac{1}{x}=\frac{-1+x}{x^2}$

$f'(1)=0$ $x=0$ is a S.P.

$f(1)=1+\ln 1=1+0=1$

$y=1.3069$ $y=1$ $y=1.6363$

$\frac{1}{2} \quad - - - \quad 1 \quad + + + + + \quad 4$

$f(x)$ has an abs. min. of $y=1$ at $x=1$, a rel. max of $y=1.3069$ at $x=\frac{1}{2}$ and a rel. max. of $y=1.6363$ at $x=4$.

16. Determine the extreme values of the function $f(x) = x^2 - 4x + 7$ Domain: $(-\infty, \infty)$

$f'(x) = 2x - 4$ $f'(x) = 0$ $0 = 2x - 4$ $x = 2$ $f'(2) = 0$

$x = 2$ is a s.p. $f(2) = 2^2 - 4(2) + 7 = 3$

- - - - $y = 3$ + + + +

2 f'

$f(x)$ has an abs. min. of $y = 3$ at $x = 2$ since $f'(x)$ changes from Neg. to Pos.

17. Determine the extreme values of the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x - 1$

$f(x)$ has a rel. min. of $y = -2\frac{1}{6}$ at $x = 1$ since f' changes from Neg. to Pos

Domain: $(-\infty, \infty)$ $f'(x) = x^2 + x - 2 = (x+2)(x-1)$

$f'(-2) = 0$ $f'(1) = 0$ $x = -2$ and $x = 1$ are s.P.

+ + + $y = 2\frac{1}{3}$ - - - $y = -2\frac{1}{6}$ + + +

-2 1 f'

$f(-2) = 2\frac{1}{3}$ $f(1) = -2\frac{1}{6}$

$f(x)$ has a rel. max. of $y = 2\frac{1}{3}$ at $x = -2$ since f' changes from Pos. to Neg.

18. Determine the extreme values of the function $f(x) = x^4 - 4x^3 + 1$

Domain: $(-\infty, \infty)$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$ $f'(0) = 0$ $f'(3) = 0$

$x = 0$ and $x = 3$ are C.P. & S.P.

$f(0) = 1$

- - - - - - - - + + + f'

0 3

$f(3) = -26$

$f(x)$ has no rel. max. or abs. max. values.

$f(x)$ has an abs. min. of $y = -26$ at $x = 3$ since $f'(x)$ changes from Neg. to Pos.